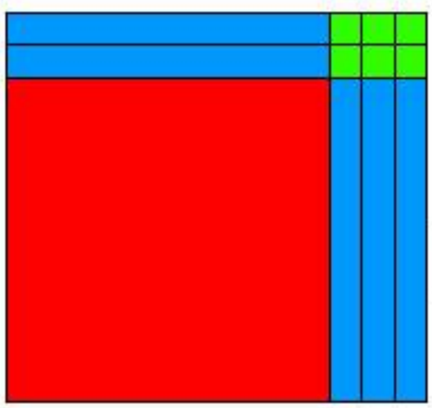
Appendix Four

Multiplication



```
      10 + 3 →
      10 + 2 ↑
     | 20 |  6
 100 | 30 |
 100+50+6
```

Introduction to Multiplication using Manipulatives

This introduction is necessary because I want to make it perfectly clear what we are trying to do here. The feedback I am getting tells me that people are looking at the attached appendix on Multiplication using manipulatives and shrugging, as if to say, “So? So you play with blocks and do sums like 2x3 = 6, so what? Just learn that 2 times 3 is 6 and you’re done. Easy. What’s the big deal about that?”

The big deal is this, for the last 100 plus years all that kids have been made to do in school to learn math is to memorize facts, rules formulae and process. There has been no attempt to achieve UNDERSTANDING. The result of this has been math anxiety, frustration, and abandonment of math as soon as possible by some students, thus no chance at a high-paying and rewarding career in Science, Technology, Engineering or Mathematics.

The source of this frustration and failure is that all the children ever saw were symbols: 1,2,3,4,5, + = - / and x. square roots, %, etc.. The thing is that ALL YOU CAN DO WITH SYMBOLS IS MEMORIZE THEM. Pages and pages of symbols, such as x^2 + 3x + 2, and rules for operations on them, all of which have to be memorized.

You cannot get to UNDERSTANDING by mere MEMORIZATION.

What we are doing in Mortensen math is making it visual, tactile, providing concrete experiences of the world by using manipulatives (plastic blocks, tiles, etc.) so the child can build concepts of how things work, without being simply told and given stuff to memorize so they can get an answer. There is a special way we do this involving seeing numbers as rectangles, counting the over and up dimensions and more, as you will see below.

“We are decoding this mathematical language, x^2 + 3x + 2, into a concrete reality so it can be understood. There is no mathematical concept that a child cannot understand if it is presented at the child’s level.” - Jerry Mortensen.

Using plastic blocks not only makes it concrete, providing tactile experiences, it also makes it visual. Now the child can SEE how the numbers work, how big they are relative to each other. In adult life, we solve real problems by visualizing solutions, by seeing what the problem is and imagining ways to solve it. We don’t just memorize a rule and an answer appears. So why teach math that way?

The appendix on multiplication shows how to put out the blocks, so that the child can SEE what six is, and how many threes it takes to make six. She can hold six in her hand and grasp concepts using all her senses and thereby develop a visceral understanding of the world through mathematics. So, put out the blocks. Build some rectangles. Count how many and start enjoying it. Let’s play math!

The Fundamentals of the Mortensen Method / Pedagogy

1) All we do in math is count
2) To count you must know what one is, so define your unit
3) SEE all numbers as rectangles, then
4) All operations are just building rectangles

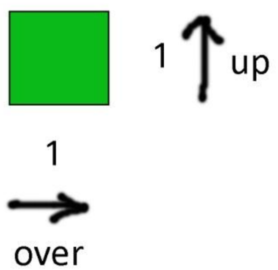
The 4-part lesson model: Build it, draw it, do notation, record answer

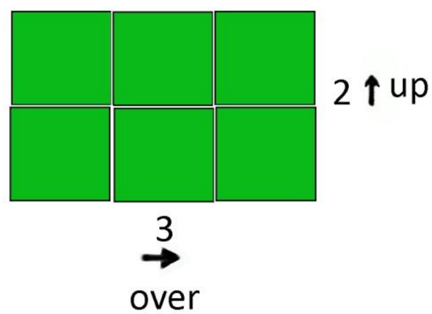
Building rectangles makes the counting faster and more accurate.
Visualize all numbers as rectangles
All you need to do math is to be able to count to nine and build rectangles
Since all we do in math is count the only question to ask is, How many?

When building numbers:
work from left to right,
from bottom upwards, and
start with the largest possible pieces
– unless otherwise indicated.

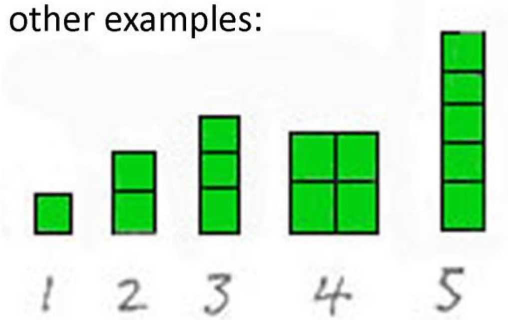
Now, use the following appendix to begin teaching multiplication.

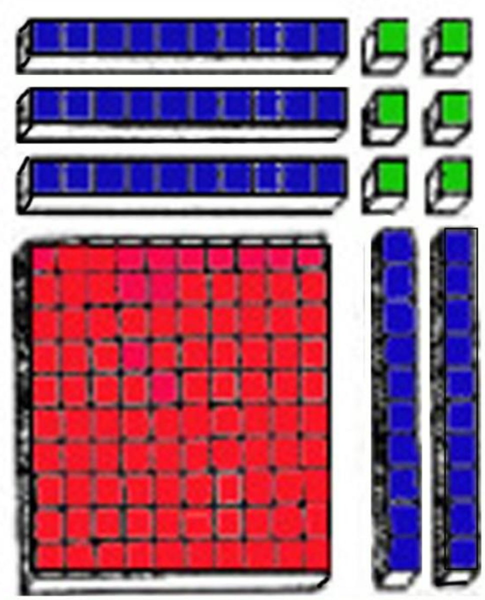
Lesson number one:
SEE all numbers as rectangles.
This, 6, is not six. It is just the name of six written as an Arabic numeral.
This is six: a rectangle, 3 over, two up, say.





other examples:





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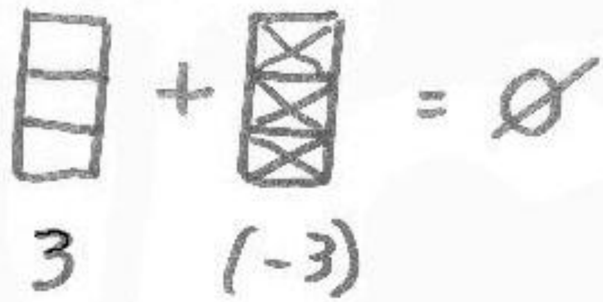
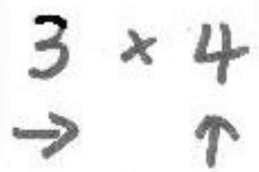
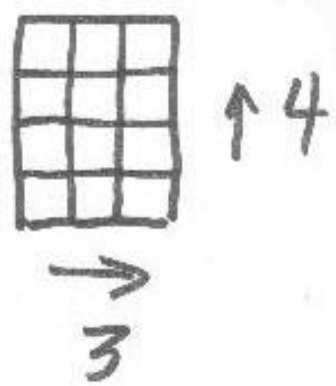
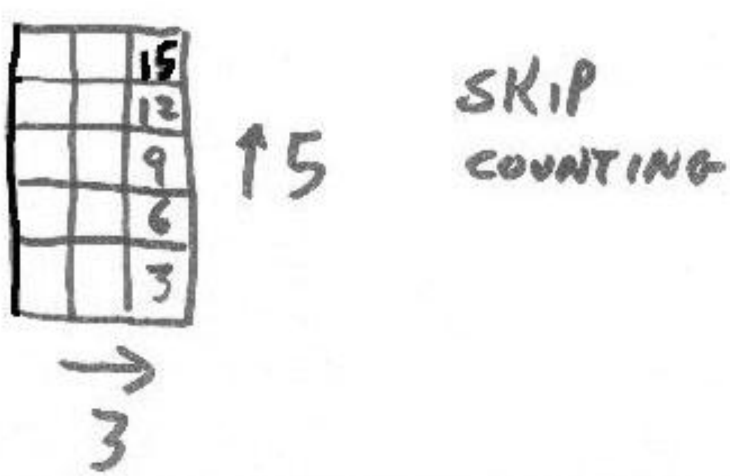




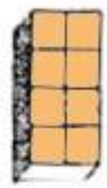
NB. In places I have used hand drawn figures because that is the way teachers and students will do it.



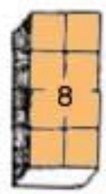


Mortensen Math
Static Multiplication

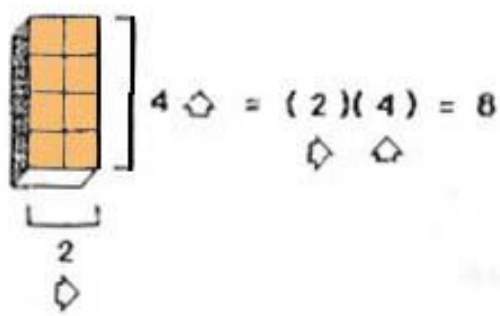
All we do in math is count. To facilitate rapid, accurate counting we build rectangles. If you see all numbers as rectangles then all operations are just building rectangles. Let's illustrate this using multiplication. First we learn to read the 'over' and 'up' dimensions of the rectangle as well as the area.
Put out four of the two blocks like this:



The area of the rectangle is 8. we can count the units to verify this.



The dimensions of the rectangle, i.e. the over and up dimensions, are 2 and 4.
Thus, 2 counted 4 times is 8. Sometimes written like this: (2)(4) = 8



We will also skip-count the rectangle going up the right hand side, saying two, four, six, eight, but more on skip-counting later

1. Static Multiplication with units

First problem: 2 X 3. Since all we do in math is count, this problem is read as "How many is 2, counted three times?"

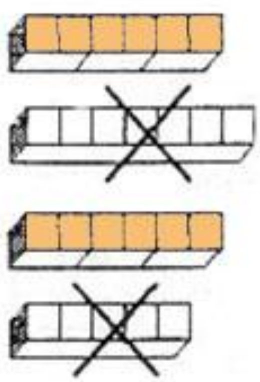
Since all numbers are seen as rectangles, and the answer is a number, let's build the rectangle. Put out three of the two blocks on the table. In doing so you have just counted two, three times. Build a rectangle starting with a two block as row one. Put the others on top, building away from you. We are not building a wall.

Start building the rectangle by putting down a two bar.

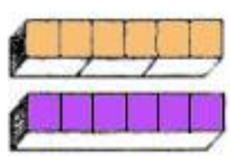
The rectangle is now two over, one up.
It needs to be two over, three up to complete the problem.
Put the next bar on top as shown below.
And the third one on top of that. The rectangle is now two over, three up.

Initially students can count the units in the model.
Then they can confirm by laying out the rectangle in a line.

Line up the two bars and compare number bars until one matches the line.
This will be a trial and success exercise.



By the explore and discover exercise, we learn that three 2 bars
is the same as a six bar, confirmed by our counting.

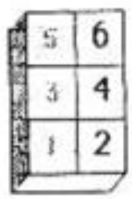


2 times 3 equals 6

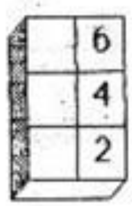
Now, regarding counting in groups, or skip-counting.
This is the best way to become familiar with multiplication facts.
instead of memorizing times tables, say.

1. All the numbers in the row have to be counted but only the last number in each row has to be counted loud. Repeat this in a faster and faster mode.

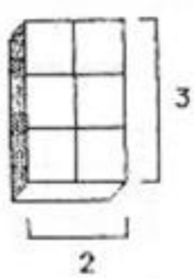
2. Whisper the numbers in the row, but shout out the last number in each row. Repeat this in a faster and faster mode.



3. Just say the last number in each row. Repeat this in a faster and faster mode.

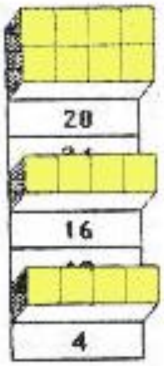


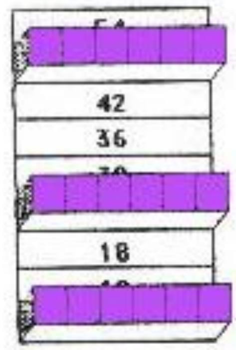
4. Solve the multiplication problem by defining the dimensions and the area of the rectangle.



$(2)(3) = 6$

Multiplication facts are the milestone in the development of the child's computational skills. The process of internalizing facts takes several months. This can be accomplished by involving the child in a variety of games and activities. You can refer to the special booklet on Games and Activities.



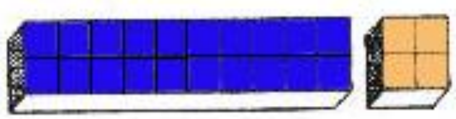


2. Static multiplication with tens and units:

Problem: 12 times 2 = **(12)(2) =** **12 x 2 =**

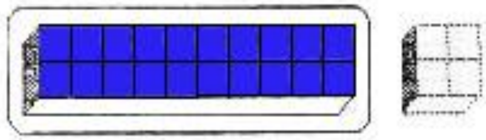
A static problem involving units and tens. The horizontal , (' over '), dimension of the re
is 12 units, and the vertical, (' up '), dimension is 2 units. Take 12, 2 unit times. Take out

Here is 12 once. To take 12 twice, construct the same rectangle once more and place it abo
first rectangle.

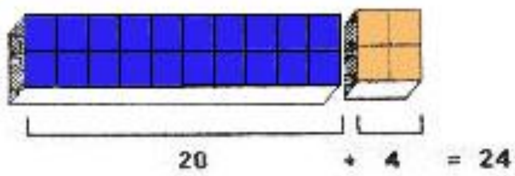


12 taken twice. Count the different values starting with the units. 4 units

and two tens:



12 times 2 is 2 tens and 4 units, 24, (twenty four).



Place value indicates that there is a place to record different categories, (Units, tens and hundreds, thousands and ten thousands).

Separate the units and tens.

See the following diagram appear.

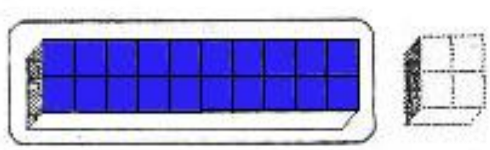
$$\begin{array}{r} 1\ 2 \\ \times\quad 2 \\ \hline \square\,\square \end{array}$$

Record the values in their own column starting with the smallest amount from right to left.

Record 4 in the unit box.

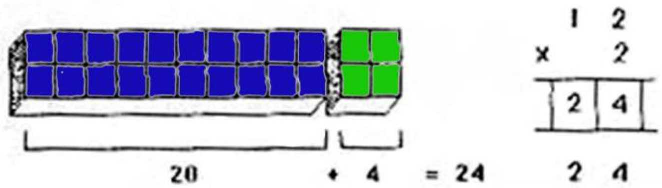
$$\begin{array}{r} 1\ 2 \\ \times\quad 2 \\ \hline \square\ 4 \end{array}$$

Record a 2 in the tens box.



$$\begin{array}{r} 1\ 2 \\ \times\quad 2 \\ \hline 2\ 4 \end{array}$$

The total amount is: 2 tens and 4 units, 24, (twenty four).



3. Static multiplication with tens and units:

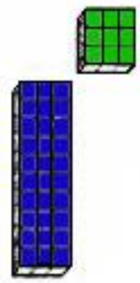
Problem: 3 times 13 = (3)(13) = 3 x 13 =

The horizontal dimension has to be 3, and the vertical dimension has to be 13. Place a ten bar in a vertical direction on the work surface.

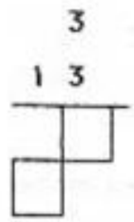
Place a 3 bar above the ten bar.

The rectangle has a horizontal dimension of 1 and a vertical dimension of 13. To obtain a horizontal dimension of 3 place the same rectangle next to the first one, two more times.

Now make two columns by shifting the tens to the left.

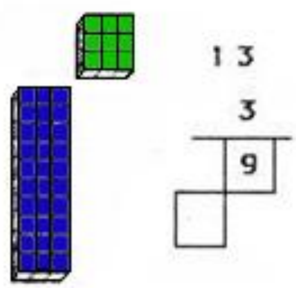


Now that we have separated the tens from the units, the notation looks like this:

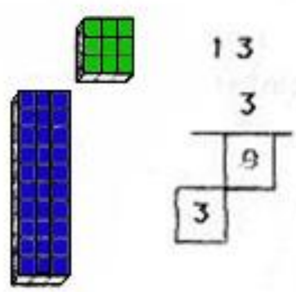


Write the partial products in the correct boxes.

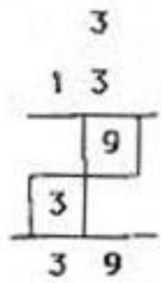
There are nine units in the top row so, write 9 in the box on the upper right.



There are three tens in the bottom row so write 3 in the tens box.



The total is: 3 tens and 9 units. Record the total amount in the spaces provided.

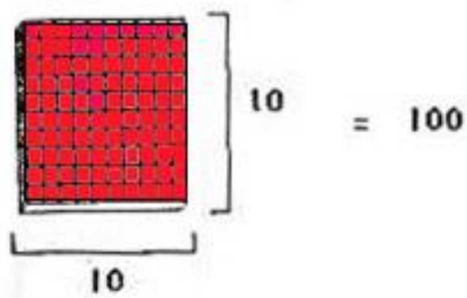


4. Static multiplication with tens and units:

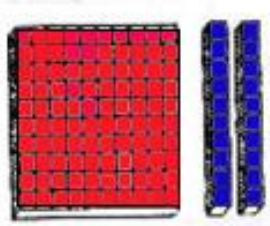
Problem: 12 times 13 = (12)(13) = 12 x 13 =

Take 12, 13 unit times. Place the biggest piece, the hundred, on the work surface.

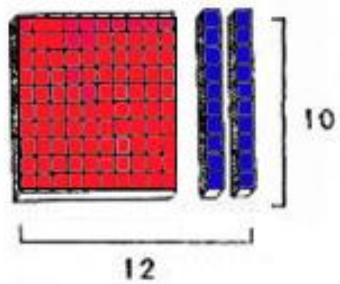
The horizontal dimension is 10 and the vertical dimension is 10.



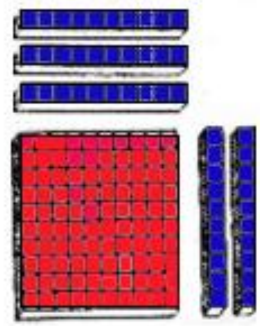
Place 2 tens on the right side of the hundred.



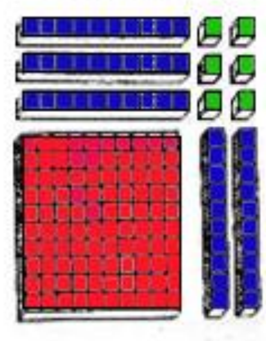
This represents 12 taken 10 times.



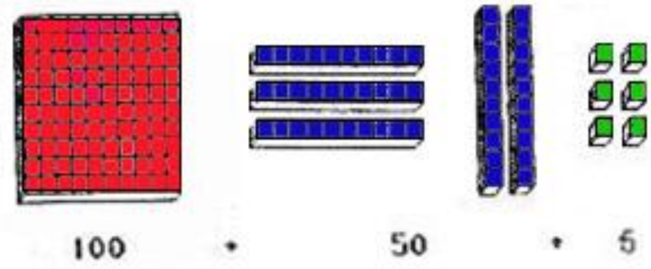
The vertical dimension which is 10 at the moment must be increased to 13. To establish a vertical dimension of 13, place 3 tens above the hundred as shown in the illustration.



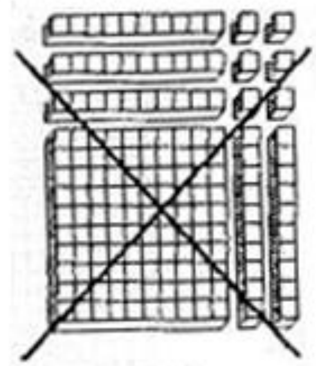
To complete the rectangle fill in the corner with 6 units.



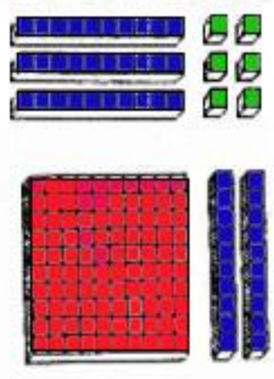
12 times 13 = 6 units, 5 tens and 1 hundred or 100 + 50 + 6 is 156, (one hundred and fifty six).



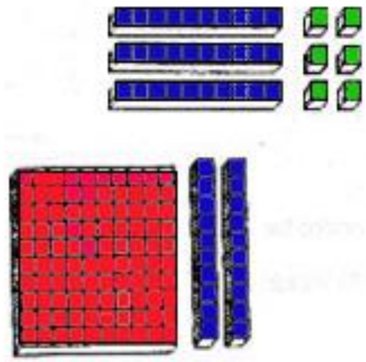
Separate the different catogories.



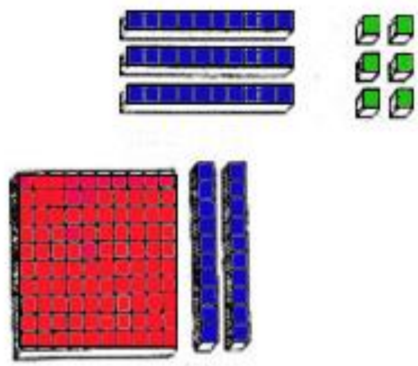
Start by separating the bottom rectangle form the top rectangle.



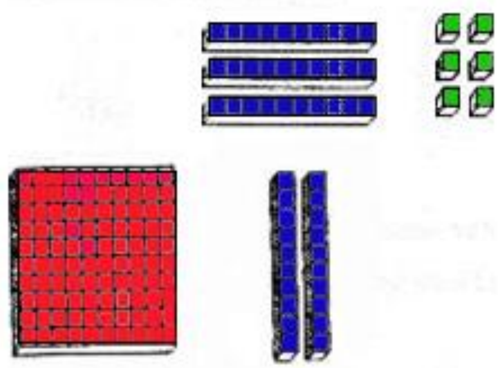
Slide the bottom rectangle to the left to line up the 10 bars.



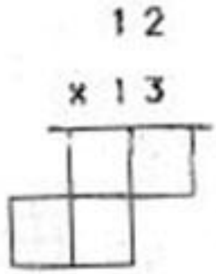
Separate the units from the ten bars in the top row.



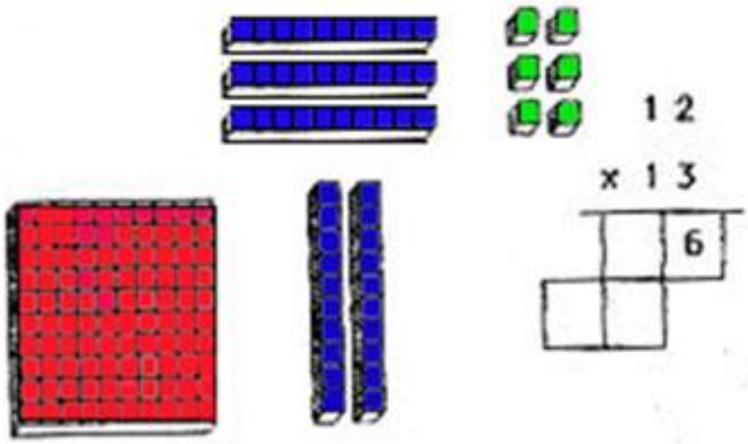
Separate the hundred from the ten bars in the bottom row.



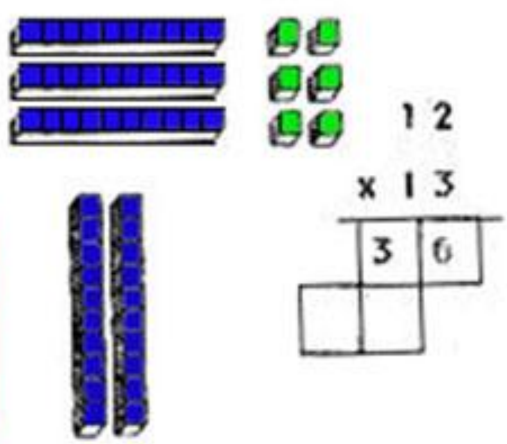
All the partial products are now lined up in their own column. See the following diagram appear.



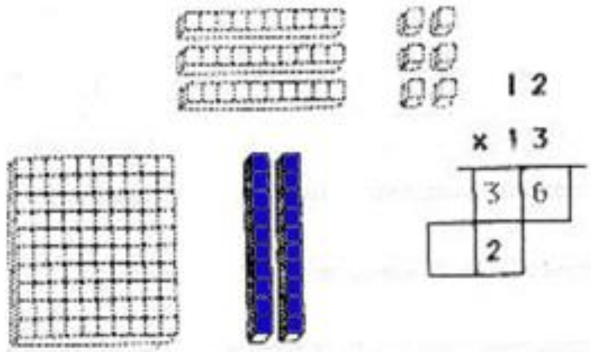
Begin recording the partial products in the boxes matching the construction. Start with the top right hand box: write "6"



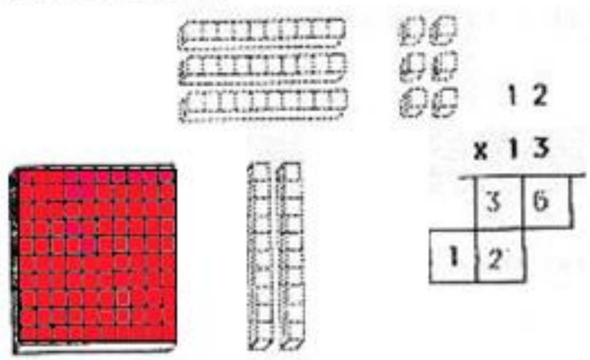
Now write 3 in the box to the left of the units, in the tens box.



Next, write 2 in the tens box below the 3.



Record 1 in the box to the left of the tens.



Record the total below the line. 1 square of 100, 5 tens and 6 units, 156, (one hundred and fifty six).

```
    1 2
  x 1 3
  -------
   [3][6]
[1][2]
  -------
  1  5  6
```

5. Static multiplication with hundreds, tens and units:

Problem: 112 times (4) = (112)(4) = 112 x 4 =

Construct a rectangle with a horizontal dimension of 100 plus 10 plus 2, and a vertical dimension of 4.

Establish a horizontal dimension of 100 by placing a reshaped hundred on the work surface.

(the strip 100 by 1).

The horizontal dimension is 100. To increase the horizontal dimension by 10, place a ten bar next to the hundred.

To increase the horizontal dimension by 2, place 2 units next to the rectangle.

To create a vertical dimension of 4 construct the same rectangle three more times and place it above the first rectangle.

Record the partial products and the total in the format illustrated below.

$$\begin{array}{r} 1\ 1\ 2 \\ \times\quad 4 \\ \hline \boxed{}\boxed{}\boxed{} \end{array}$$

Separate the different categories.

Record 8 in the right hand box.

$$\begin{array}{r} 1\ 1\ 2 \\ \times\quad 4 \\ \hline \boxed{}\boxed{}\boxed{8} \end{array}$$

Record a 4 in the box next to the units

	1	1 2
x		4
	4	8

Record 4 in the box next to the tens.

1	1	2
x		4
4	4	8

Record the total below the line. The total is: 4 hundreds, 4 tens and 8 units, 448, (four hundred and forty eight)

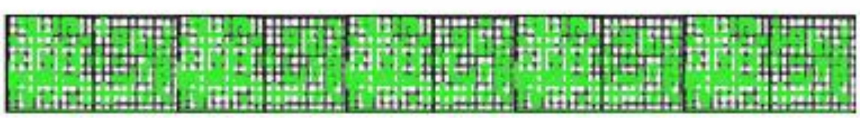
1	1	2
x		4
4	4	8
4	4	8

6. Static multiplication with hundreds, tens and units:

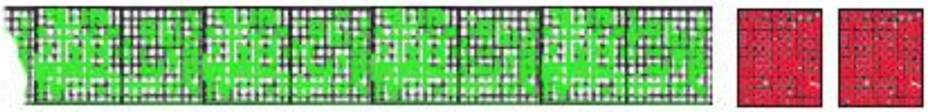
Problem: 122 times 22 = (122)(22) = **122 x 22 =**

Construct a rectangle with a horizontal dimension of 100 plus 20 plus 2, and a vertical dimension of 20 + 2

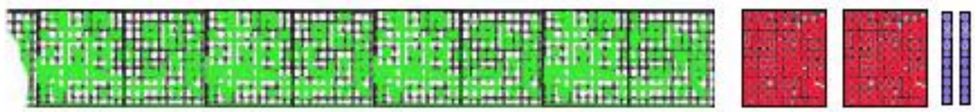
Establish a horizontal dimension of 100 by placing the biggest piece, (the thousand bar) on the work surface.



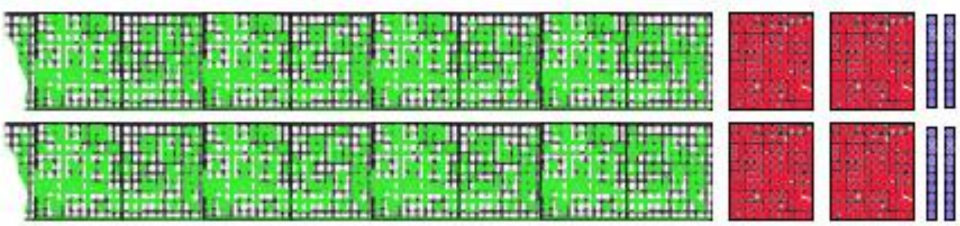
The horizontal dimension is 100. To increase the horizontal dimension by 20, place 2 hundreds next to the thousand.



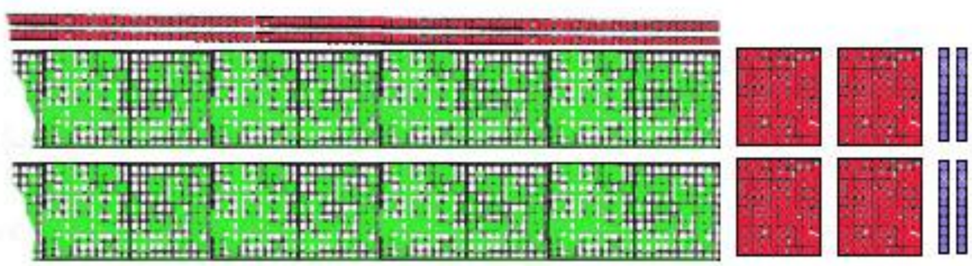
To increase the horizontal dimension by 2, place 2 tens next to the rectangle.



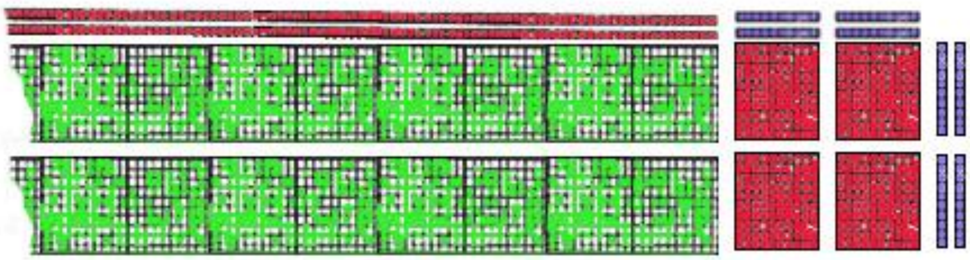
To create a vertical dimension of 22 first construct a vertical dimension of 20 by building the same rectangle once more and placing it above the first rectangle.



Increase the vertical dimension by 2 by placing 2 strips of one hundred above the thousand bars.

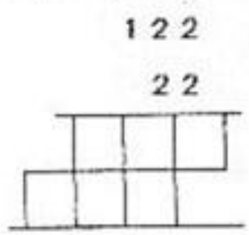


Place 2 tens next to each strip of 100, which calls for 4 tens.

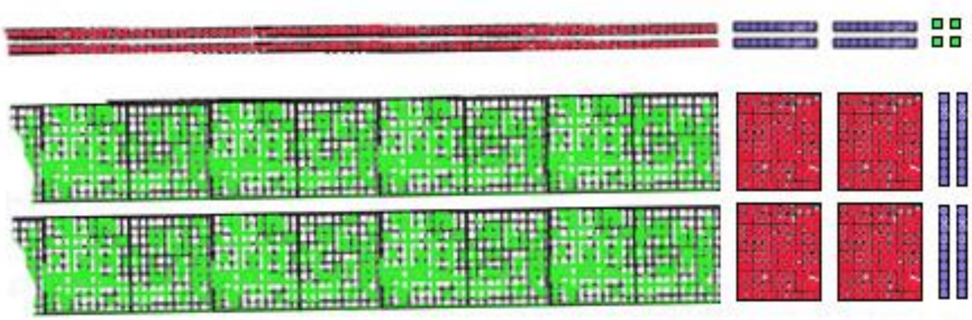


To complete the rectangle fill in the corner with 4 units.

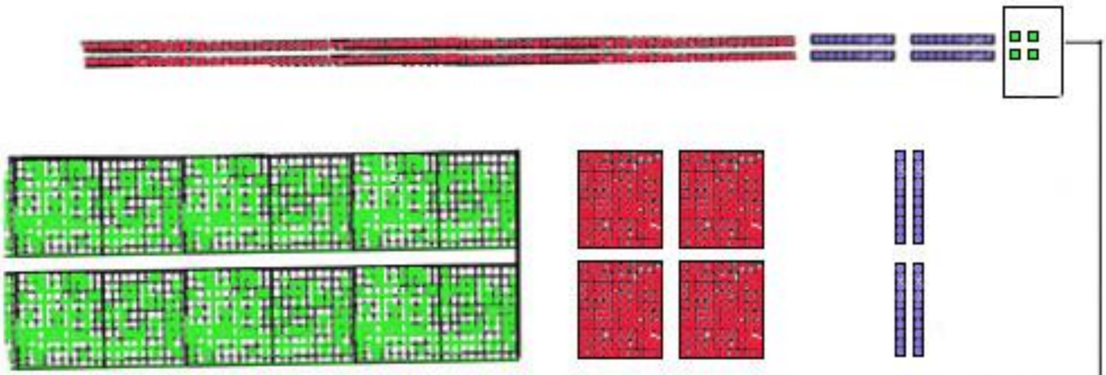
To complete the problem record the partial products and the total in the format illustrated below.



Separate the bottom rectangle from the top rectangle.



Separate all the partial products from each other until they are all lined up in their own column.

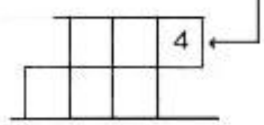


1 2 2

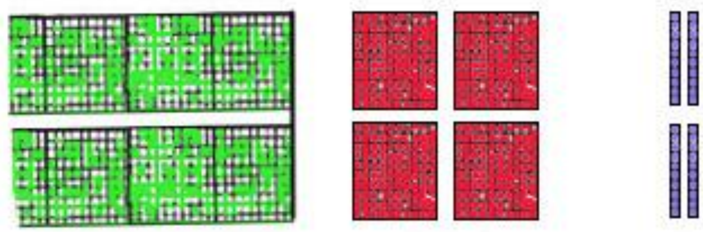
x 2 2

Start by recording the units first.

Put a 4 in the box below the 2s

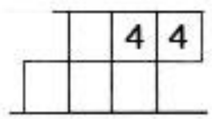


Write 4 in the tens box next door:

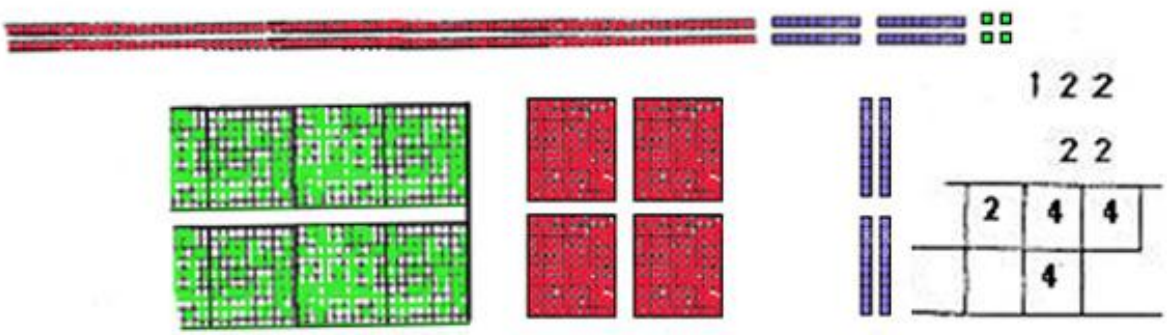


1 2 2

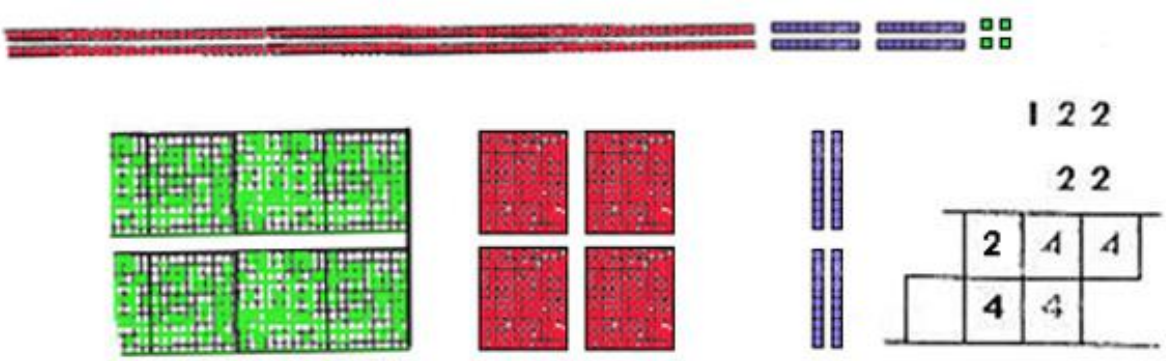
x 2 2



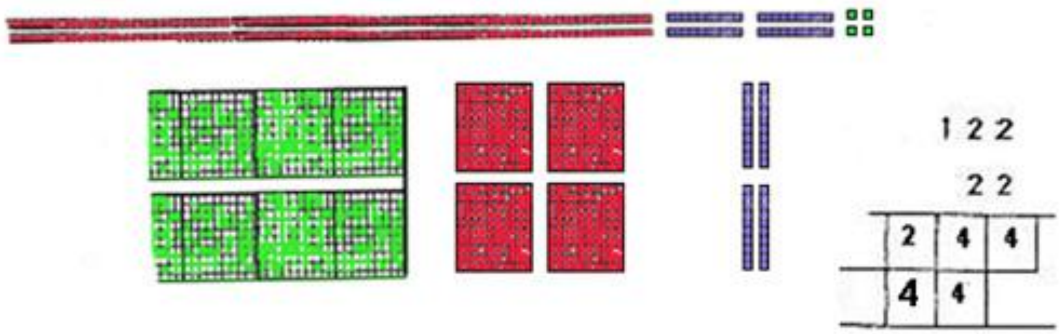
Move to the bottom rectangle. Record 4 in the bottom right hand box.

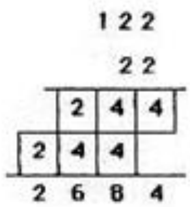


Record a 2 in the box next to the tens



Record a 2 in the box next to the hundreds





Sum each column to the bottom line:
2 thousands, 6 hundreds, 8 tens, 4 units or
two thousand six hundred, eighty-four